Problem 1: Prototype Link [+20 pts]

You are testing a prototype link interconnecting a CPU and high-speed memory with a length of 1 cm and for an operating signal frequency of 100 GHz. The link inductance is $L' = 25$ nH/m, and its capacitance is $C' = 1$ nF/m. Determine whether this link needs to be modeled as a transmission line. Provide a rationale.

The factors that determine whether or not we should treat the wires as a transmission line are governed by the length of the line and the frequency of the signal provided by the generator (i.e., $\frac{L}{\lambda} \geq 0.01$ where λ can be calculated from $u_p = f\lambda$.

Assuming a lossless transmission line:

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{25 \times 10^{-9} \times 1 \times 10^{-9}}} = 2 \times 10^8 [\text{m/s}]$$

$$\frac{L}{\lambda} = \frac{L}{u_p/f} = \frac{1 \times 10^{-2}}{2 \times 10^8/(100 \times 10^9)} = 5 \leq 0.01$$

The link needs to be modeled as a transmission line as the phase shift due to time delay needs to be accounted for.

Problem 2: Characteristic Impedance [+30 pts]

Consider a transmission line where both the magnitude and phase of the characteristic impedance Z_0 are known. The voltage or current propagating on the transmission line are <u>not known</u>.

a **Explain what information can be derived from Z_0 about the magnitude and phase of the voltage and current on the line.**

No information can be derived about V_0^+, V_0^-, I_0^+, or I_0^- since they all rely on information about the load and generator. However, since we know Z_0, we can get expressions that relate the forward and backward propagating waves.

$$\begin{aligned} I_0^+ = \frac{V_0^+}{Z_0} \Longrightarrow |I_0^+| = \frac{|V_0^+|}{|Z_0|} \quad \text{and} \quad \angle I_0^+ = \angle V_0^+ - \angle Z_0 \\ I_0^- = -\frac{V_0^-}{Z_0} \Longrightarrow |I_0^-| = \frac{|V_0^-|}{|Z_0|} \quad \text{and} \quad \angle I_0^- = \pm\pi + \angle V_0^- - \angle Z_0 \end{aligned} \tag{1}$$

b **If the imaginary component of Z_0 is nonzero ($Im\{Z_0\} \neq 0$), what can be determined about the phase relationship between the voltage and current.**

If $Im\{Z_0\} \neq 0$, then we know it will have an associated non-zero phase angle, $\angle Z_0$. Using equations 1, we can see that I_0^+ and V_0^+ are <u>not</u> in phase and I_0^- and V_0^- are <u>not</u> 180^o (or π) out of phase.

c **If $Im\{Z_0\} \neq 0$, what can be determined about the loss properties of the line.**

Consider using the equation for Z_0, equation 2, and knowing that for a lossless line $R' = 0$ and $G' = 0$.

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \tag{2}$$

$$Z_0 = \sqrt{\frac{j\omega L'}{j\omega C'}} = \sqrt{\frac{L'}{C'}}$$

which is a <u>real</u> number. We can conclude that for a lossless line, $Im\{Z_0\} = 0$. Therefore, if $Im\{Z_0\} \neq 0$, the line must have loss. NOTE: However, for a lossy line, where $R' \neq 0$ and $G' \neq 0$, let us consider a case in which the numerator and denominator of equation 2 are equal, resulting in a Z_0 of 1 Ω. Clearly, the imaginary part of this Z_0 is zero. Therefore, we <u>cannot</u> conclude the reverse, that for a lossy line, $Im\{Z_0\} \neq 0$.

Problem 3: Voltage and Current on Transmission Lines [+30 pts]

Consider a transmission line with a characteristic impedance $Z_0 = 100\ \Omega$ ($Im\{Z_0\} = 0$). At a given position ($z = z'$) on the line, the voltage is measured to be $1V$ ($|\tilde{V}(z')| = 1V$).

(a) **Determine what is known about the *total* current on the line**

We have no information about the load, so if $Z_L \neq Z_0$ (impedance mismatch), then we cannot calculate $\tilde{I}(z)$. If $Z_L = Z_0$ (matched), then

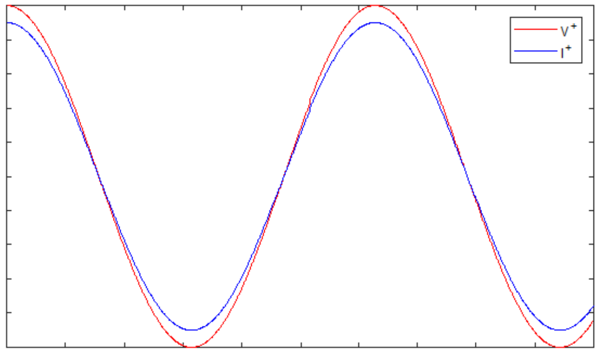
$$\tilde{I}(z') = \frac{\tilde{V}(z')}{Z_0} \Longrightarrow |\tilde{I}(z')| = \frac{1V}{100\Omega} = 10mA$$

(b) **Sketch the forward-traveling voltage and current waves on the transmission line at the snapshot in time, showing their amplitude and phases.**

Z_0 is real ($\angle Z_0 = 0$), so we can use our relations from equations 1.

$$\angle I_0^+ = \angle V_0^+ - \angle Z_0 \Longrightarrow \angle I_0^+ = \angle V_0^+$$

So I_0^+ is in phase with V_0^+.

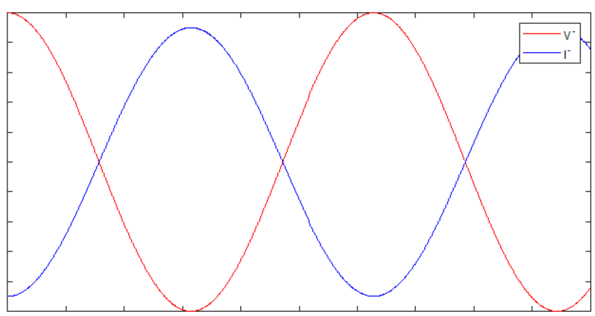


Without knowledge of the generator or load, we cannot determine the magnitudes of the voltage and current, so phase is the important part of the sketch. Choice of z' is arbitrary.

(c) **Sketch the backward-traveling voltage and current waves on the transmission line at a snapshot in time, showing their amplitudes and phases.**
Similarly,

$$\angle I_0^- = \pm\pi + \angle V_0^- - \angle Z_0 \Longrightarrow \angle I_0^- = \pm\pi + \angle V_0^-$$

So I_0^- is 180^o (or π) out of phase with V_0^-.



Problem 4: Voltage and Current on Transmission Lines [+20 pts]

For each of the following characteristics of standing waves on a lossless short-circuited line, find the frequency of the source exciting the line:

a **The distance between successive nodes of the voltage amplitude is 50 cm and the dielectric is air.**

We know that for a short-circuited line, the distance between successive nodes of a standing wave is half the wavelength of the source. So, $\lambda/2 = 50$ cm, or $\lambda = 100$ cm. For a lossless air line, the wave propagates at approximately the speed of light in vacuum. Therefore, we have

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{1 \text{ m}} = 300 \text{ MHz}$$

b **The distance between successive nodes of the current amplitude is 50 cm and the dielectric is nonmagnetic with $\epsilon = 9\epsilon_0$.**

We still have a wavelength of 100 cm, however we now have a different dielectric. A non-magnetic dielectric has the permeability of free space. This particular dielectric has a relative permittivity of 9 times that of free space. This changes the phase velocity to

$$u_p = \frac{c}{\sqrt{\epsilon}} = \frac{c}{\sqrt{\frac{9\epsilon_0}{\epsilon_0}}} = 10^8 \text{ m/s}$$

resulting in

$$f = \frac{u_p}{\lambda} = 100 \text{ MHz}$$